PHYS 101 – General Physics I Midterm Exam Solution

1. Show your work and write your answer for each part in the box provided.

(a) A point object (particle 1), which is at the origin at time $t = 0$, has initial velocity $\vec{\mathbf{v}}_{10} = -16\,\hat{\mathbf{i}} - 4\hat{\mathbf{j}}$ (m/s) and constant acceleration $\vec{\mathbf{a}}_1 = 4\,\hat{\mathbf{i}} + \hat{\mathbf{j}}$ (m/s^2).

i) (5 Pts.) Find the position $\vec{\mathbf{r}}_1$ where particle 1 comes to rest (momentarily).

$$\vec{\mathbf{r}}_1 = \vec{\mathbf{r}}_{10} + \vec{\mathbf{v}}_{10}t + \tfrac{1}{2}\vec{\mathbf{a}}_1 t^2 \quad \rightarrow \quad \vec{\mathbf{r}}_1 = (-16\,t + 2\,t^2)\hat{\mathbf{i}} + \left(-4\,t + \tfrac{1}{2}t^2\right)\hat{\mathbf{j}}\ (\mathrm{m})$$

$$\boxed{\vec{\mathbf{r}}_1(4) = -32\,\hat{\mathbf{i}} - 8\,\hat{\mathbf{j}}\ (\mathrm{m})}$$

$$\vec{\mathbf{v}}_1 = \frac{d\vec{\mathbf{r}}_1}{dt} = (-16 + 4\,\mathrm{t})\hat{\mathbf{i}} + (-4 + t)\hat{\mathbf{j}}\ (\mathrm{m/s})$$

$$\vec{\mathbf{v}}_1 = 0 \quad \rightarrow \quad -16 + 4\,\mathrm{t} = 0, \qquad -4 + t = 0 \quad \rightarrow \quad t = 4\ \mathrm{s} \quad \rightarrow \quad \vec{\mathbf{r}}_1(4) = -32\,\hat{\mathbf{i}} - 8\,\hat{\mathbf{j}}\ (\mathrm{m})$$

ii) (5 Pts.) What is the displacement vector of the particle 1 during the first two seconds?

$$\vec{\mathbf{r}}_1(2) = -24\,\hat{\mathbf{i}} - 6\,\hat{\mathbf{j}}\ (\mathrm{m}),\ \ \vec{\mathbf{r}}_1(0) = 0$$

$$\Delta\vec{\mathbf{r}}_1 = \vec{\mathbf{r}}_1(2) - \vec{\mathbf{r}}_1(0) = -24\,\hat{\mathbf{i}} - 6\,\hat{\mathbf{j}}\ (\mathrm{m})$$

$$\boxed{\Delta\vec{\mathbf{r}}_1 = -24\,\hat{\mathbf{i}} - 6\,\hat{\mathbf{j}}\ (\mathrm{m})}$$

iii) (5 Pts.) What is the average velocity of the particle 1 during the first two seconds?

$$\vec{\mathbf{v}}_{1\ \mathrm{Av}} = \frac{\Delta\vec{\mathbf{r}}_1}{\Delta t} = -12\,\hat{\mathbf{i}} - 3\,\hat{\mathbf{j}}\ (\mathrm{m/s})$$

$$\boxed{\vec{\mathbf{v}}_{1\ \mathrm{Av}} = -12\,\hat{\mathbf{i}} - 3\,\hat{\mathbf{j}}\ (\mathrm{m/s})}$$

(b) The position of another point object (particle 2) is given by $\vec{\mathbf{r}}_2 = (-1 + 2\,t)\hat{\mathbf{i}} + (2 - t^2)\hat{\mathbf{k}}$ (m).

iv) (5 Pts.) What is the acceleration of particle 2?

$$\vec{\mathbf{v}}_2 = \frac{d\vec{\mathbf{r}}_2}{dt} = 2\,\hat{\mathbf{i}} - 2\,t\,\hat{\mathbf{k}}\ (\mathrm{m/s}) \quad \rightarrow \quad \vec{\mathbf{a}}_2 = \frac{d\vec{\mathbf{v}}_2}{dt} = -2\,\hat{\mathbf{k}}\ (\mathrm{m/s^2})$$

$$\boxed{\vec{\mathbf{a}}_2 = -2\,\hat{\mathbf{k}}\ (\mathrm{m/s^2})}$$

v) (5 Pts.) What is the relative velocity of particle 2 with respect to particle 1 as a function of time?

$$\vec{\mathbf{v}}_{2/1} = \vec{\mathbf{v}}_{2/E} + \vec{\mathbf{v}}_{E/1} = \vec{\mathbf{v}}_{2/E} - \vec{\mathbf{v}}_{1/E}$$

$$\vec{\mathbf{v}}_{2/1} = 2\,\hat{\mathbf{i}} - 2\,t\,\hat{\mathbf{k}} - [(-16 + 4\,\mathrm{t})\hat{\mathbf{i}} + (-4 + t)\hat{\mathbf{j}}]\ (\mathrm{m/s})$$

$$\vec{\mathbf{v}}_{2/1} = (18 - 4\,\mathrm{t})\hat{\mathbf{i}} + (4 - t)\hat{\mathbf{j}} - (2\,t)\hat{\mathbf{k}}\ (\mathrm{m/s})$$

$$\boxed{\vec{\mathbf{v}}_{2/1} = (18 - 4\,\mathrm{t})\hat{\mathbf{i}} + (4 - t)\hat{\mathbf{j}} - (2\,t)\hat{\mathbf{k}}\ (\mathrm{m/s})}$$

vi) (5 Pts.) What is the distance between the two particles at time $t = 2$ s?

$$\vec{\mathbf{r}}_2(2) = 3\,\hat{\mathbf{i}} - 2\,\hat{\mathbf{k}}\ (\mathrm{m})$$

$$d = |\vec{\mathbf{r}}_2(2) - \vec{\mathbf{r}}_1(2)| = \left|27\,\hat{\mathbf{i}} + 6\,\hat{\mathbf{j}} - 2\,\hat{\mathbf{k}}\right|\ (\mathrm{m})$$

$$d = \sqrt{(27)^2 + (6)^2 + (2)^2} = \sqrt{769}\ (\mathrm{m})$$

$$\boxed{d = \sqrt{769}\ (\mathrm{m})}$$

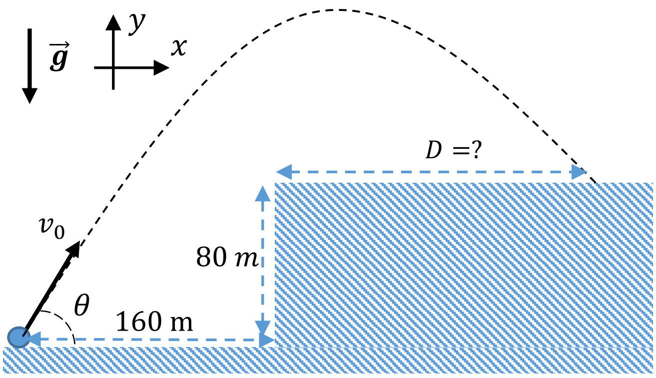
2. A projectile is launched from the ground with initial speed $v_0 = 62.5$ m/s at an angle θ with the horizontal. Given that $\sin\theta = 4/5$. There is a vertical cliff at a distance 160 m away from the launching point which rises by a distance 80 m. Gravitational acceleration is $g = 10\ m/s^2$.

(a) (5 Pts.) What is the maximum height from the ground level reached by the projectile?

(b) (10 Pts.) What is the total time of flight?

(c) (10 Pts.) How far away from the edge of the cliff does the ball fall (i.e., $D = ?$)?

(d) (10 Pts.) For the given angle θ, what is the smallest initial speed that can get the ball to the top of the step?



Solution: (a)

$$x = v_0 \cos\theta\ t\,, \qquad y = v_0 \sin\theta\ t - \frac{1}{2} g\, t^2 \quad \rightarrow \quad x = (37.5)\, t\ (\text{m})\,, \qquad y = 50\, t - 5\, t^2\ (\text{m}).$$

$$v_y = \frac{dy}{dt} = 50 - 10\, t\ (\text{m/s})\,, \qquad v_y = 0 \quad \rightarrow \quad t_{\max} = 5\ \text{s} \quad \rightarrow \quad y_{\max} = y(t_{\max}) = 125\ (\text{m}).$$

(b)

$$y = 80\ (m) \quad \rightarrow \quad 80 = 50\, t - 5\, t^2 \quad \rightarrow \quad t^2 - 10\, t + 16 = 0 \quad \rightarrow \quad t_1 = 2\ (\text{s})\,, \qquad t_2 = 8\ (\text{s})$$

$$t_{\text{tot}} = t_2 = 8\ (\text{s})$$

(c)

$$x(8) = (37.5)\, 8 = 300\ (\text{m}) \quad \rightarrow \quad \text{D} = 300 - 160 = 140\ (\text{m})$$

(d)

$$x = v_0 \cos\theta\ t\,, \qquad y = v_0 \sin\theta\ t - \frac{1}{2} g\, t^2 \quad \rightarrow \quad y = x \tan\theta - \frac{g\, x^2}{2 v_0^2 (\cos\theta)^2}$$

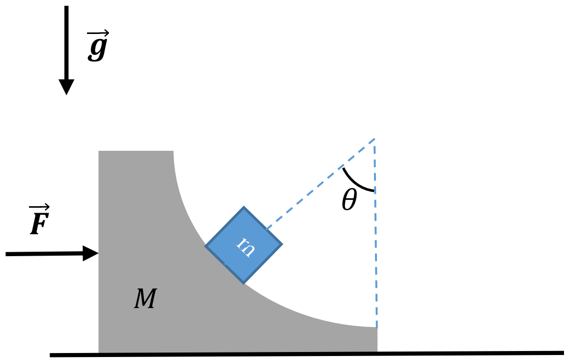
$$y(x = 160\ \text{m}) = 80\ \text{m} \quad \rightarrow \quad v_0 = 40\sqrt{\frac{5}{3}}\ (\text{m/s}) \cong 51.6\ (\text{m/s}).$$

3. A ramp of mass M is free to slide on a frictionless plane. A small object with mass m is on the part of the ramp which is frictionless and shaped like a quarter circle. When M is acted on by a horizontal force $\vec{F}$, both masses move together, and the small mass goes to an equilibrium position at angle θ as shown in the figure.

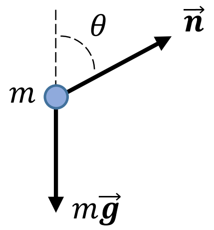
(a) (10 Pts.) Draw the free body diagrams for both masses.

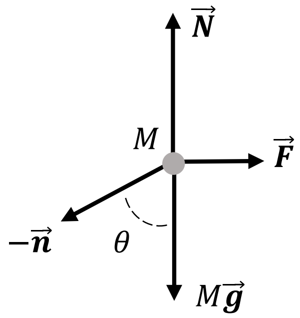
(b) (15 Pts.) Find the magnitude F of the force in terms of M, m, θ and the gravitational acceleration g.

(c) (10 Pts.) Find the magnitude N of the normal force the floor is applying to M, in terms of of M, m, θ and g.



Solution: (a)





(b) Newton's second law implies:

$n \sin\theta = m\, a$

$n \cos\theta - mg = 0$

$F - n \sin\theta = M\, a$

$N - n\cos\theta - Mg = 0$

$$n \sin\theta = m\, a, \qquad n\cos\theta = mg \;\rightarrow\; \tan\theta = \frac{a}{g} \;\rightarrow\; a = g\tan\theta$$

$$n \sin\theta = m\, a, \qquad F - n\sin\theta = M\, a \;\rightarrow\; F = (m + M)\, a \;\rightarrow\; F = (m + M)\, g \tan\theta$$

(c)

$$n\cos\theta - mg = 0 \;\rightarrow\; n = \frac{m\, g}{\cos\theta}, \qquad N = n\cos\theta + Mg \;\rightarrow\; N = (m + M)\, g.$$